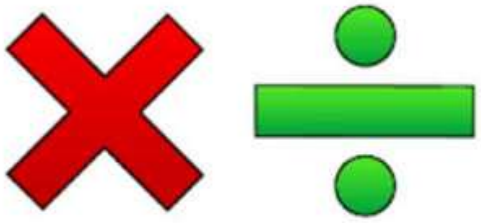
Havannah First School
Calculation Policy 2023

Challenge, Equality & Opportunity



Havannah First School Calculation Policy

Introduction

This document is a statement of the aims, benefits and skills required for the teaching and learning of calculation in Mathematics at Havannah First School.

Developmental Aims:

- To introduce children to the processes of calculation through practical, oral and mental activities.
- To support children in developing ways of recording to support their thinking and calculation methods
- Enable children to learn to interpret and use the signs and symbols.
- To facilitate children's use of models and images, such as number lines and Numicon to support their mental and informal written methods of calculation.
- To enable children to strengthen and refine their mental methods in order to develop informal written methods.
- To support children in becoming more efficient and succinct in their recordings which will ultimately lead to efficient written methods that can be used more generally.
- By the end of Year 4 children should be equipped with mental and written methods that they understand and can use correctly.
- By the end of Year 4, when faced with a calculation, children will be able to decide which method is most appropriate and have strategies to check its accuracy.
- At whatever stage in their learning, and whatever method is being used, children's methods of calculating will be underpinned by a secure and appropriate knowledge of number facts, along with the mental skills that are needed to carry out the process and judge if it was successful.

The overall aims when children leave first school are for them to:

- have a secure understanding of mental maths facts to apply to written mathematics;
- have a secure knowledge of number facts and a **good** understanding of the four operations
- have an efficient, reliable, compact written method of calculation for addition, subtraction, multiplication and division that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- be able to use this knowledge and understanding to solve problems;
- be able to recall quickly, fluently and accurately the multiplication tables up to 12 X12.

The aim is that by the end of Year 4, the great majority of children should be able to use an efficient written method for addition, subtraction and multiplication and division with confidence and understanding. Children will develop the ability to use what are commonly known as 'standard' written methods - methods that are efficient and work for any calculations. They are compact and consequently help children to keep track of their recorded steps. Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation mentally or do not have access to a calculator. We want children to know that they have such a reliable, written method to which they can turn when the need arises.

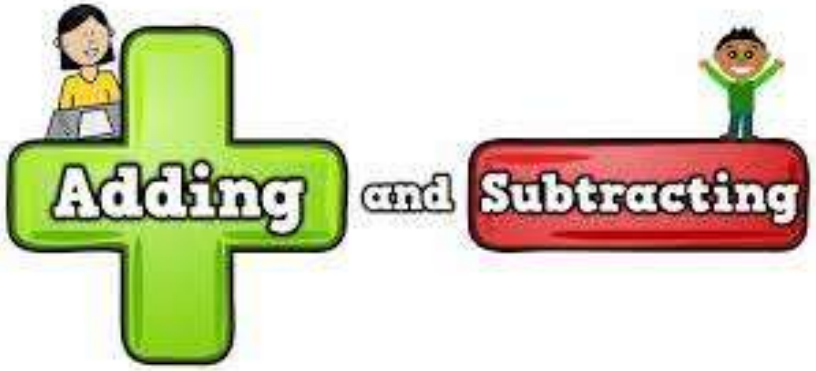
In setting out these aims, the intention is that there will be a consistent approach to the learning of calculation strategies and that all teachers understand the progression of skills and key concepts. The great majority of children will benefit greatly from learning how to use the most efficient methods. The challenge for teachers will be in determining when their children should move on to a refinement in the method and become confident and more efficient at written calculation.

Calculation Policy Document

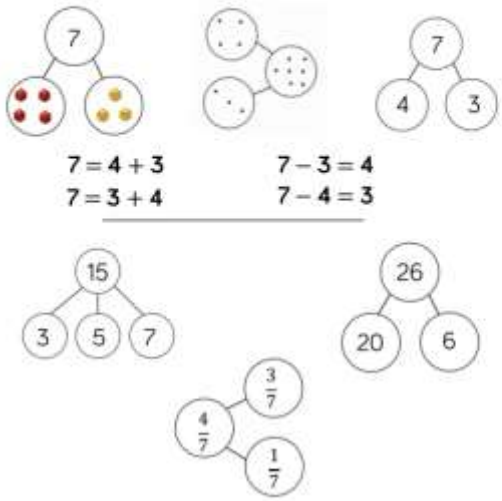
Our Calculation policy document is broken down into addition and subtraction and multiplication and division. The policy is based on and adapted from the White Rose Maths Schemes and calculation policies which are used across Havannah First School. At the start of each section, there is an overview of the different models and images that we use to support the teaching of different concepts. These provide explanations of the benefits of using the models and show the links between different operations.

Each operation is then broken down into skills and each skill has a dedicated section showing the different models and images that are used to effectively teach that concept. There is an overview of skills linked to year groups to support consistency throughout school. A glossary of terms is provided at the end of each calculation to support understanding of the key language used to teach the four operations.

ADDITION AND SUBTRACTION



Part-Whole Model



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

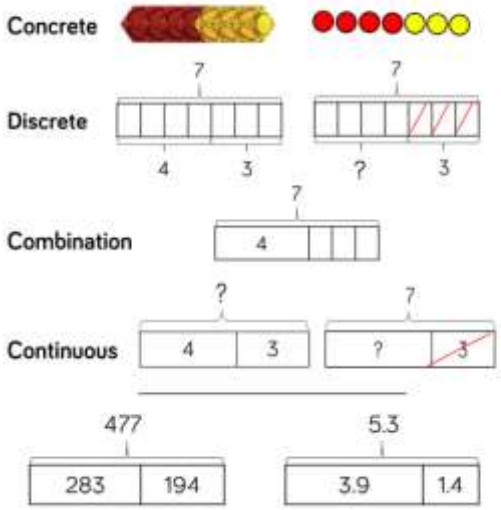
When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Bar Model (single)



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

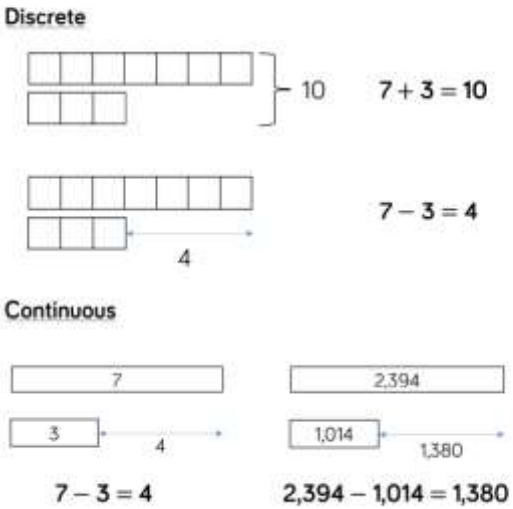
Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Bar Model (multiple)



Benefits

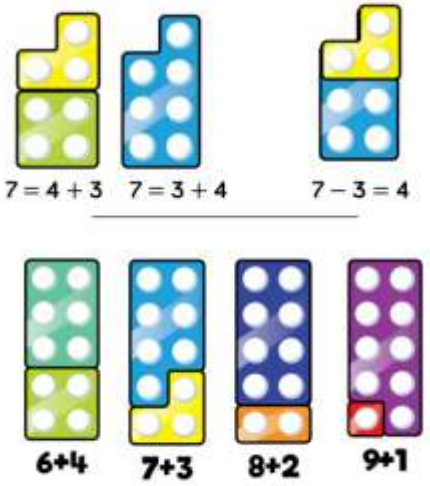
The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Number Shapes



Benefits

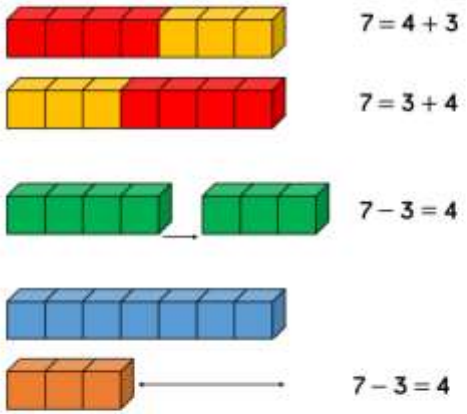
Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

Cubes



Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

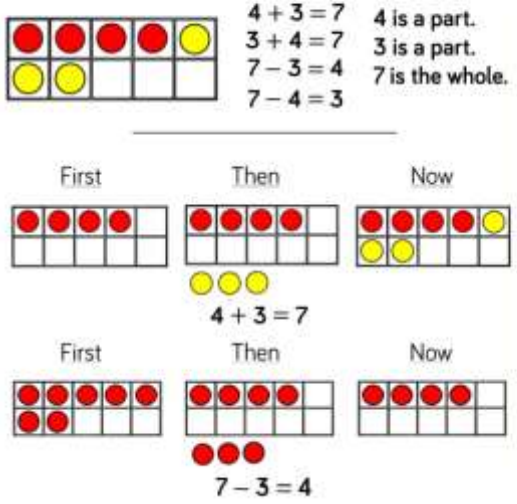
When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

Ten Frames (within 10)



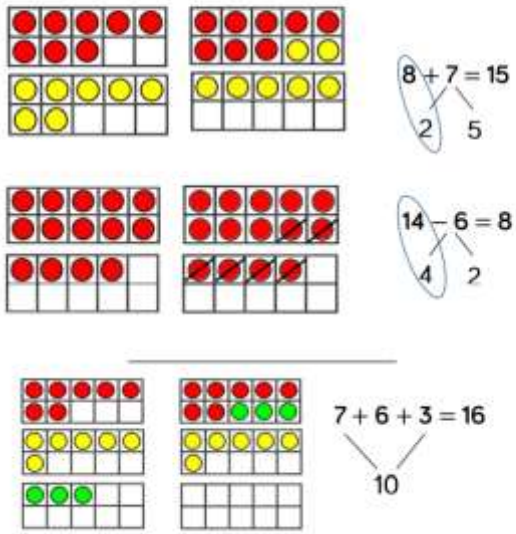
Benefits

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.
Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

Ten Frames (within 20)



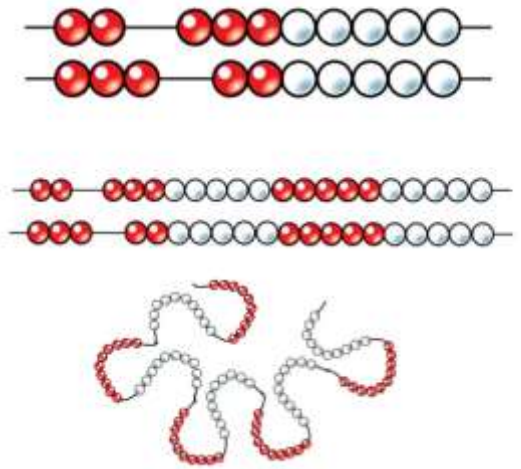
Benefits

When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

Bead Strings



Benefits

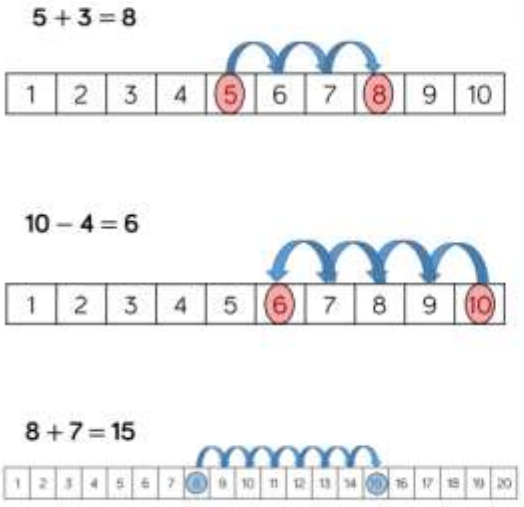
Different sizes of bead strings can support children at different stages of addition and subtraction.

Bead strings to 10 are very effective at helping children to investigate number bonds up to 10.
They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. 2 + 8 = 10, move one bead, 3 + 7 = 10.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

Number Tracks



Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

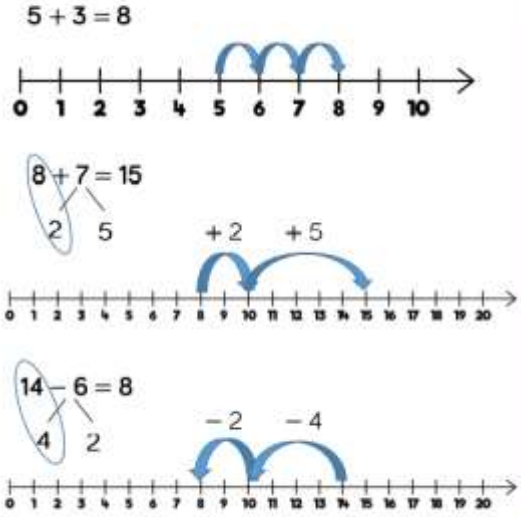
When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

Number Lines (labelled)



Benefits

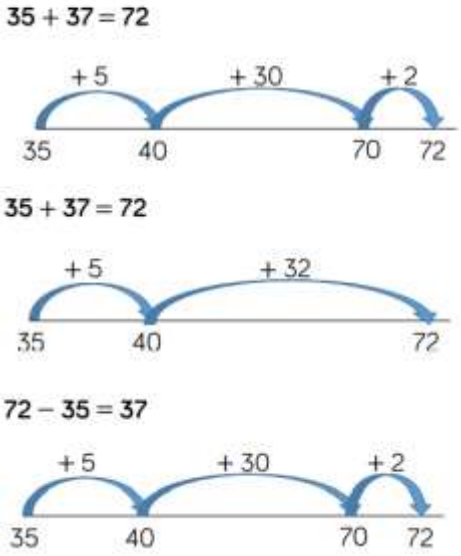
Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

Number Lines (blank)



Benefits

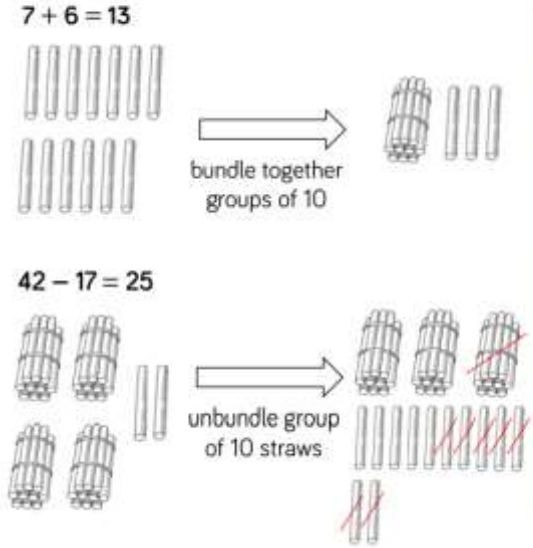
Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Straws



Benefits

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

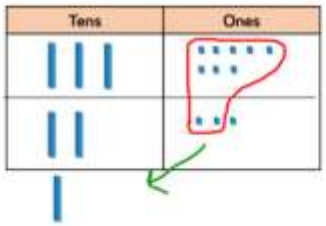
Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

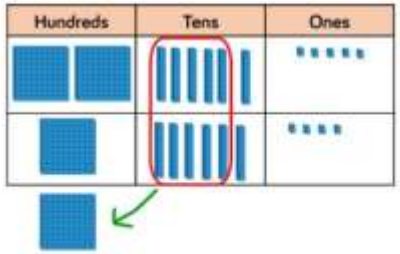
When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.

Base 10/Dienes (addition)



$$\begin{array}{r} 38 \\ +\ 23 \\ \hline 61 \\ \hline 1 \end{array}$$



$$\begin{array}{r} 265 \\ +\ 164 \\ \hline 429 \\ \hline 1 \end{array}$$

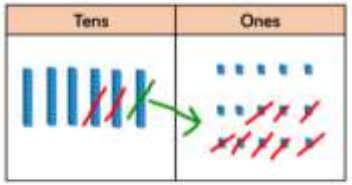
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

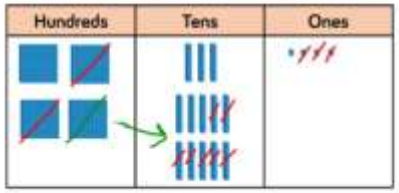
Children should first add without an exchange before moving on to addition with exchange. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children.
How many ones are there altogether?
Can we make an exchange? (Yes or No)
How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)
How many ones do we have left? (Write in ones column)
Repeat for each column.

Base 10/Dienes (subtraction)



$$\begin{array}{r} {}^{5}\not{6}{}^{1}5 \\ -\ 28 \\ \hline 37 \end{array}$$



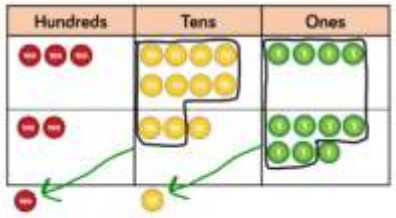
$$\begin{array}{r} {}^{3}\not{4}{}^{1}35 \\ -\ 273 \\ \hline 262 \end{array}$$

Benefits

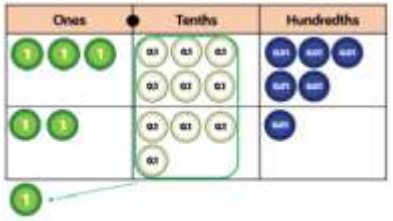
Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.
This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

Place Value Counters (addition)



$$\begin{array}{r} 384 \\ +\ 237 \\ \hline 621 \\ \hline 1\ 1 \end{array}$$



$$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$

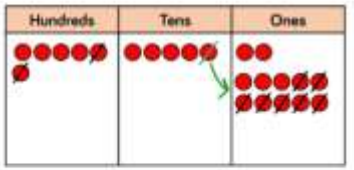
Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

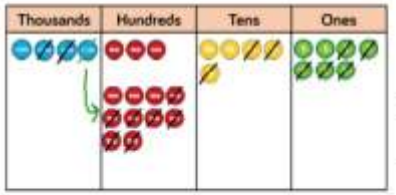
Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Place Value Counters (Subtraction)



$$\begin{array}{r} {}^{4}\not{6}{}^{1}\not{5}2 \\ -\ 207 \\ \hline 445 \end{array}$$



$$\begin{array}{r} {}^{3}\not{4}{}^{1}357 \\ -\ 2735 \\ \hline 1622 \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

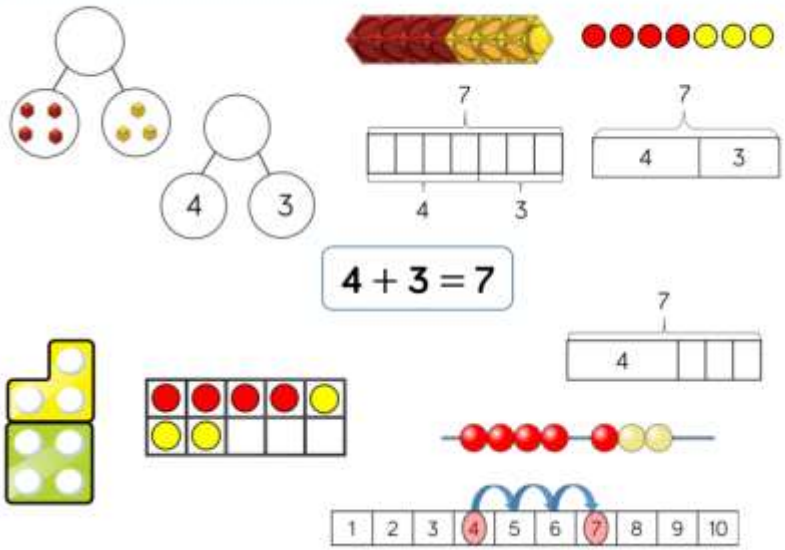
ADDITION

Skill	Year	Representations and models	
Add two 1-digit numbers to 10	1	Part-whole model Bar model Number shapes	Ten frames (within 10) Bead strings (10) Number tracks
Add 1 and 2-digit numbers to 20	1	Part-whole model Bar model Number shapes Ten frames (within 20)	Bead strings (20) Number tracks Number lines (labelled) Straws
Add three 1-digit numbers	2	Part-whole model Bar model	Ten frames (within 20) Number shapes
Add 1 and 2-digit numbers to 100	2	Part-whole model Bar model Number lines (labelled)	Number lines (blank) Straws Hundred square

Skill	Year	Representations and models	
Add two 2-digit numbers	2	Part-whole model Bar model Number lines (blank) Straws	Base 10 Place value counters Column addition
Add with up to 3-digits	3	Part-whole model Bar model	Base 10 Place value counters Column addition
Add with up to 4-digits	4	Part-whole model Bar model	Base 10 Place value counters Column addition

Skill: Add 1-digit numbers within 10

Year: 1



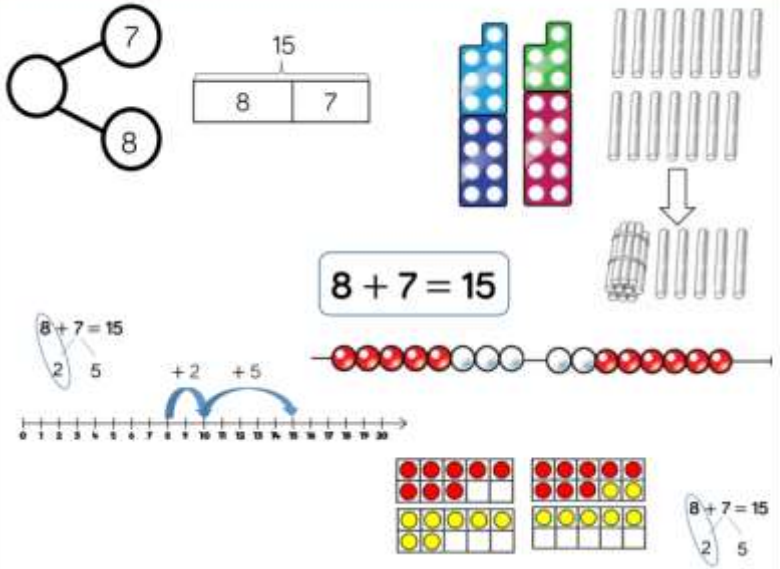
When adding numbers to 10, children can explore both aggregation and augmentation.

The part-whole model, discrete and continuous bar model, number shapes and ten frame support aggregation.

The combination bar model, ten frame, bead string and number track all support augmentation.

Skill: Add 1 and 2-digit numbers to 20

Year: 1/2

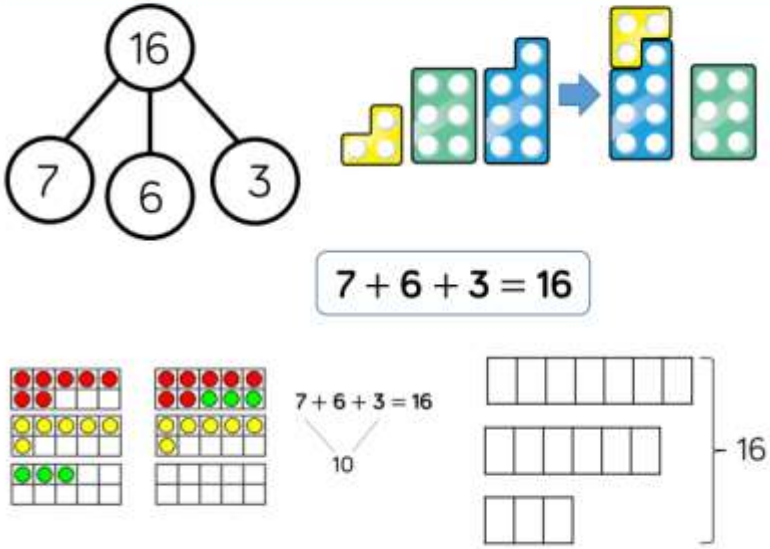


When adding one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten.

Different manipulatives can be used to represent this exchange. Use concrete resources alongside number lines to support children in understanding how to partition their jumps.

Skill: Add three 1-digit numbers

Year: 2



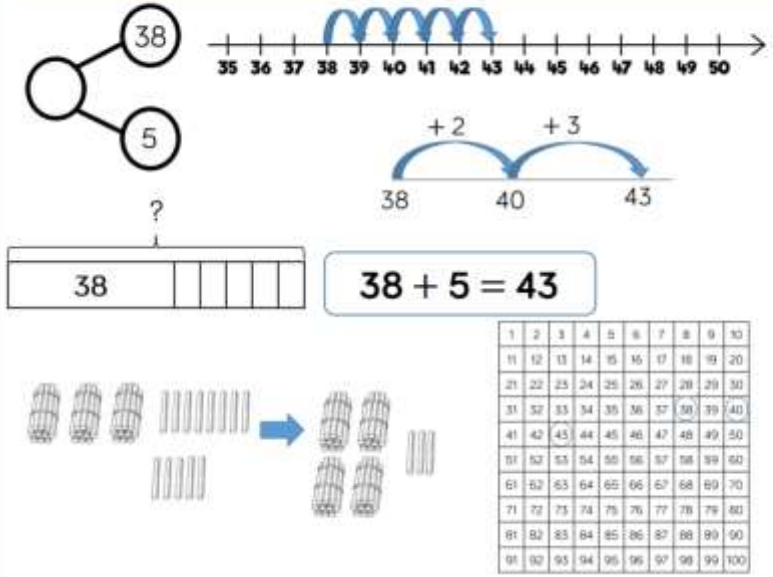
When adding three 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently.

This supports children in their understanding of commutativity.

Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers.

Skill: Add 1-digit and 2-digit numbers to 100

Year: 2/3



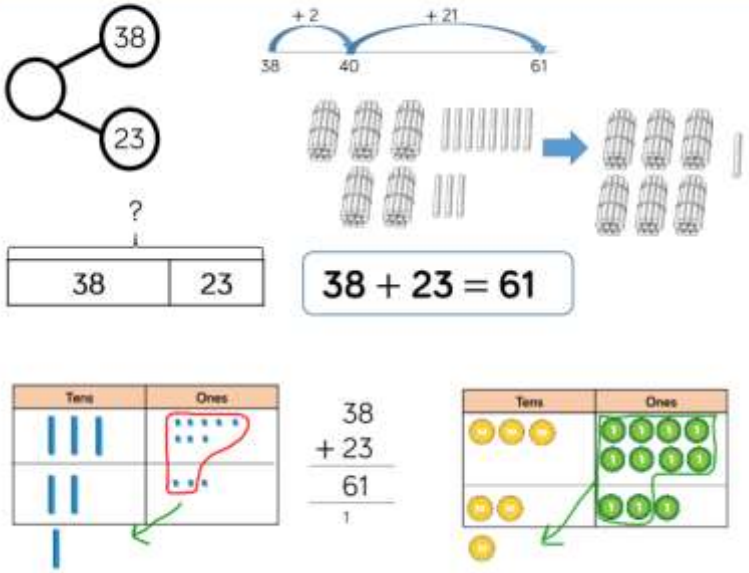
When adding single digits to a two-digit number, children should be encouraged to count on from the larger number.

They should also apply their knowledge of number bonds to add more efficiently e.g. 8 + 5 = 13 so 38 + 5 = 43.

Hundred squares and straws can support children to find the number bond to 10.

Skill: Add two 2-digit numbers to 100

Year: 2/3

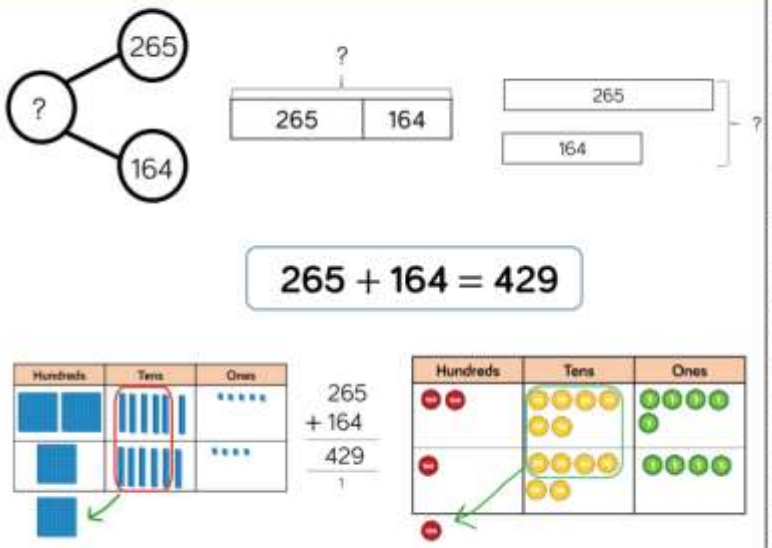


At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

Children can also use a blank number line to count on to find the total. Encourage them to jump to multiples of 10 to become more efficient.

Skill: Add numbers with up to 3 digits

Year: 3



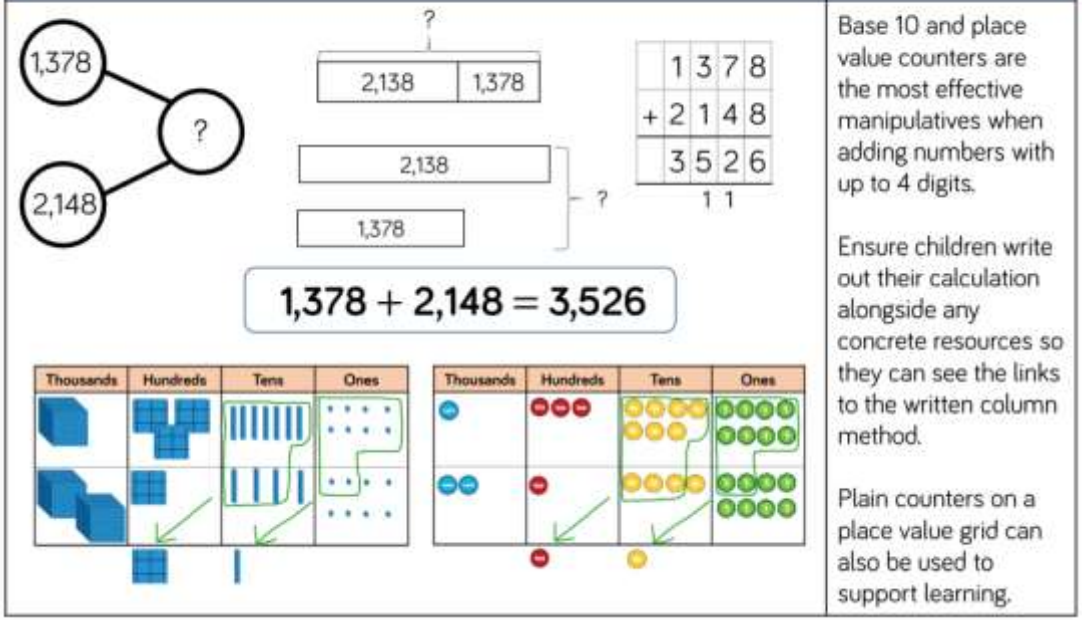
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

Skill: Add numbers with up to 4 digits

Year: 4



Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

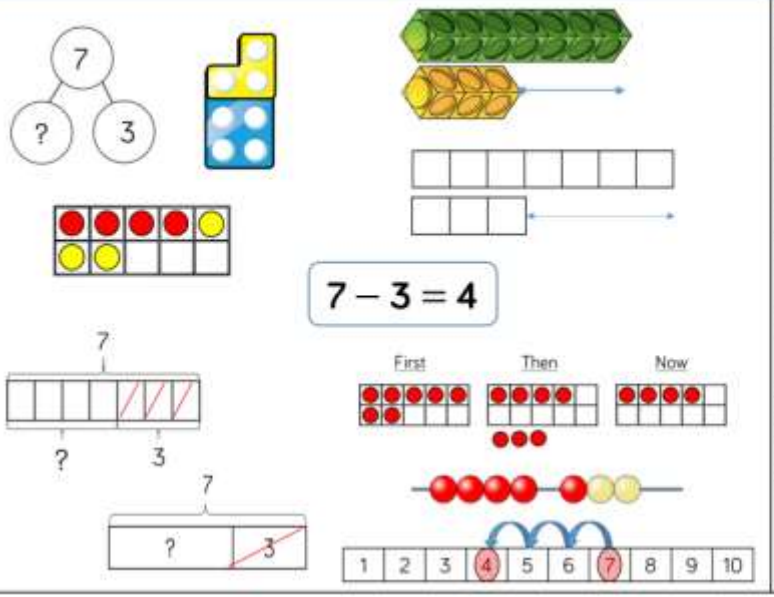
SUBTRACTION

Skill	Year	Representations and models	
Subtract two 1-digit numbers to 10	1	Part-whole model Bar model Number shapes	Ten frames (within 10) Bead strings (10) Number tracks
Subtract 1 and 2-digit numbers to 20	1	Part-whole model Bar model Number shapes Ten frames (within 20)	Bead string (20) Number tracks Number lines (labelled) Straws
Subtract 1 and 2-digit numbers to 100	2	Part-whole model Bar model Number lines (labelled)	Number lines (blank) Straws Hundred square
Subtract two 2-digit numbers	2	Part-whole model Bar model Number lines (blank) Straws	Base 10 Place value counters Column subtraction

Skill	Year	Representations and models	
Subtract with up to 3-digits	3	Part-whole model Bar model	Base 10 Place value counters Column subtraction
Subtract with up to 4-digits	4	Part-whole model Bar model	Base 10 Place value counters Column subtraction

Skill: Subtract 1-digit numbers within 10

Year: 1



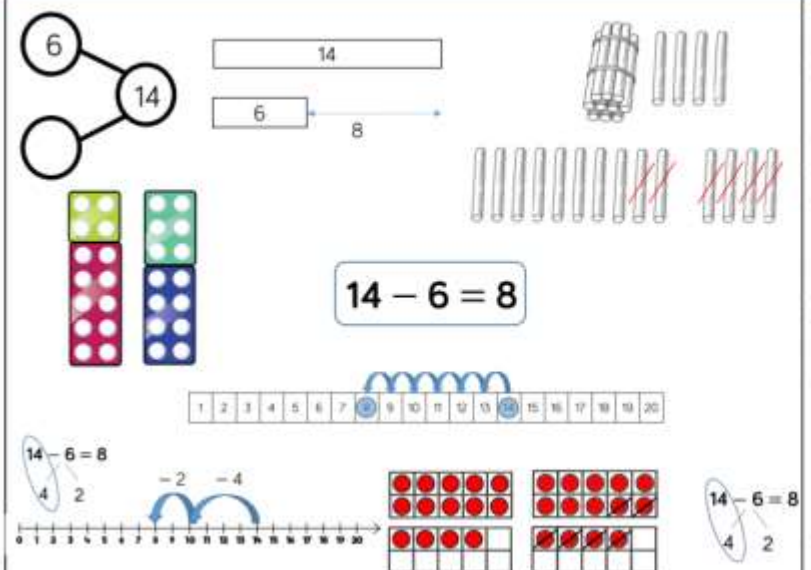
Part-whole models, bar models, ten frames and number shapes support partitioning.

Ten frames, number tracks, single bar models and bead strings support reduction.

Cubes and bar models with two bars can support finding the difference.

Skill: Subtract 1 and 2-digit numbers to 20

Year: 1/2

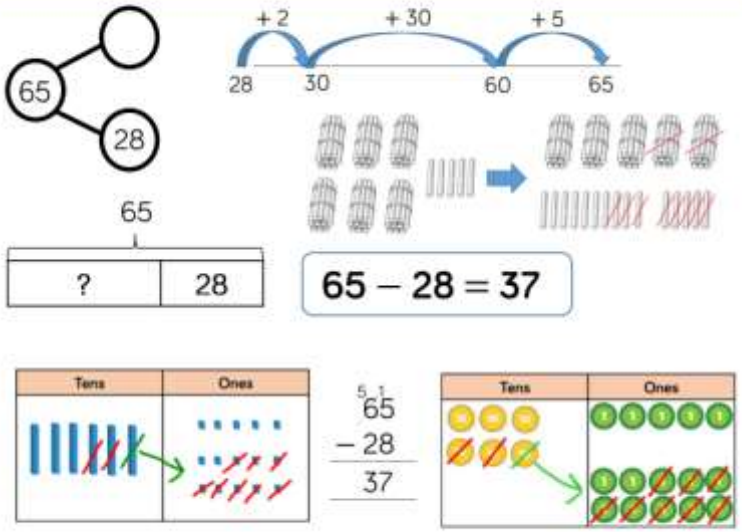


When subtracting one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten.

Children should be encouraged to find the number bond to 10 when partitioning the subtracted number. Ten frames, number shapes and number lines are particularly useful for this.

Skill: Subtract 1 and 2-digit numbers to 100

Year: 2

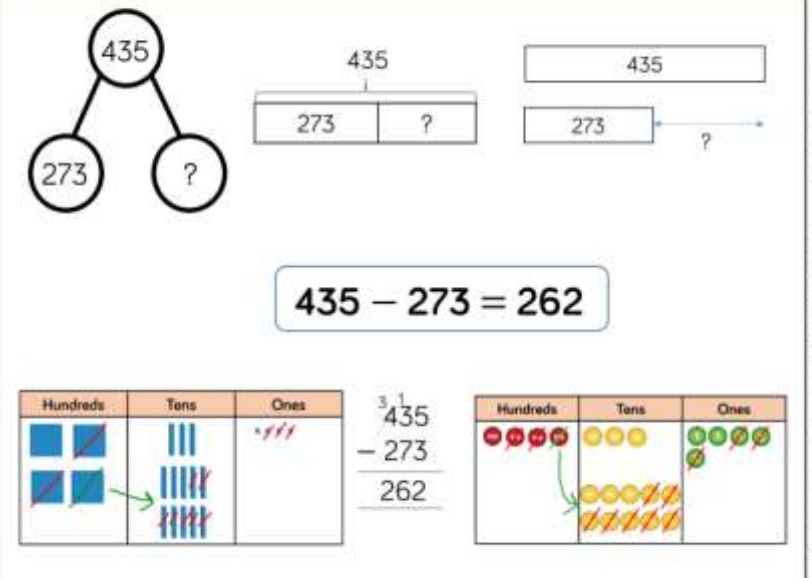


At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

Children can also use a blank number line to count on to find the difference. Encourage them to jump to multiples of 10 to become more efficient.

Skill: Subtract numbers with up to 3 digits

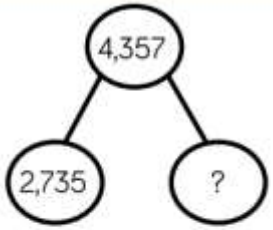
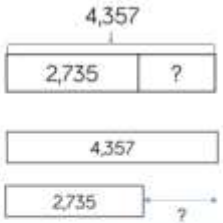
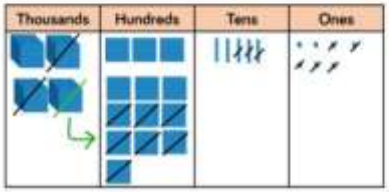
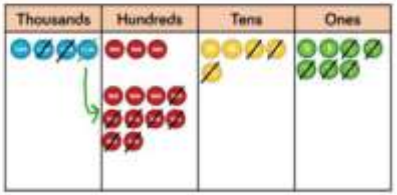
Year: 3



Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

Skill: Subtract numbers with up to 4 digits	Year: 4
  3 1 4357 − 2735 1622 **4,357 − 2,735 = 1,622**  	Base 10 and place value counters are the most effective manipulatives when subtracting numbers with up to 4 digits. Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method. Plain counters on a place value grid can also be used to support learning.

Glossary

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative – numbers can be added in any order.

Complement – in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference – the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange – Change a number or expression for another of an equal value.

Minuend – A quantity or number from which another is subtracted.

Partitioning – Splitting a number into its component parts.

Reduction – Subtraction as take away.

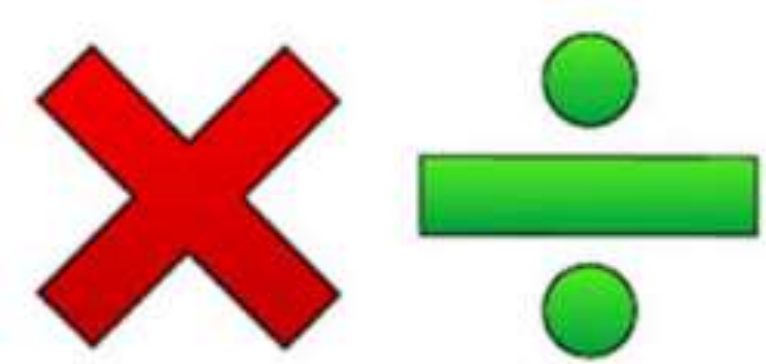
Subitise – Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

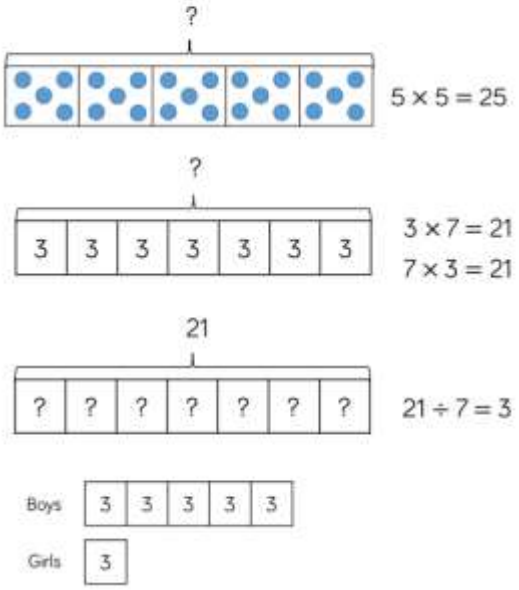
Sum - The result of an addition.

Total – The aggregate or the sum found by addition.

MULTIPLICATION AND DIVISION



Bar Model



Benefits

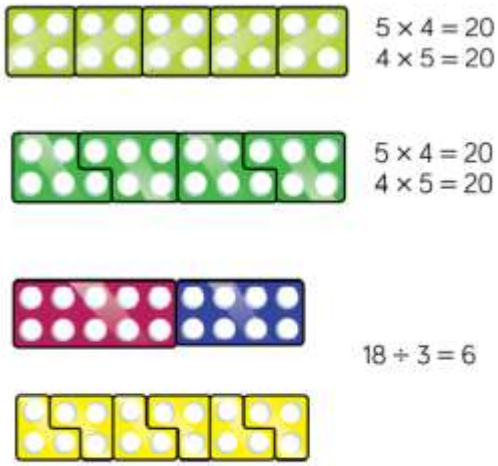
Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?
The multiple bar model provides an opportunity to compare the groups.

Number Shapes



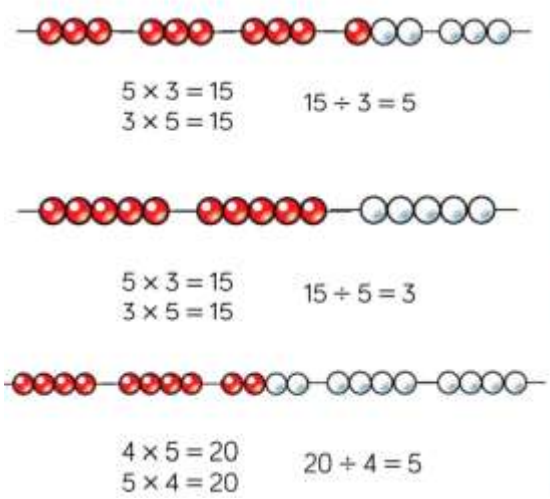
Benefits

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd × odd = even, odd × even = odd, even × even = even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.

Bead Strings



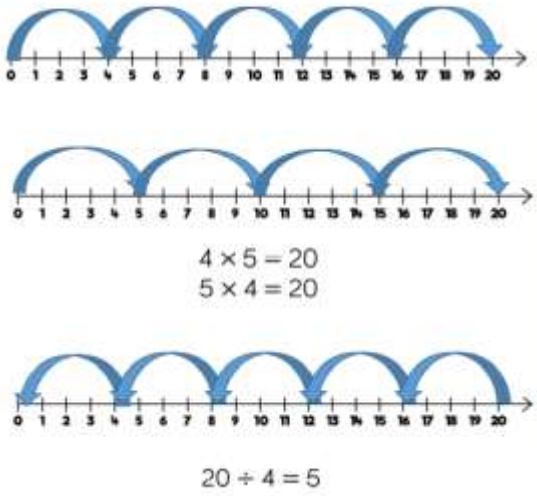
Benefits

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.
Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 – Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

Number Lines (labelled)



Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

When multiplying, children start at 0 and then count on to find the product of the numbers.
When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.
Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

Number Tracks

$6 \times 3 = 18$
$3 \times 6 = 18$

$18 \div 3 = 6$

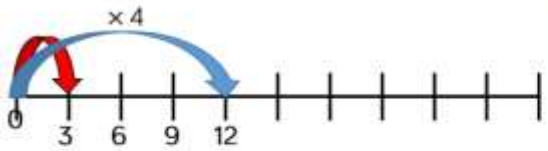
Benefits

Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

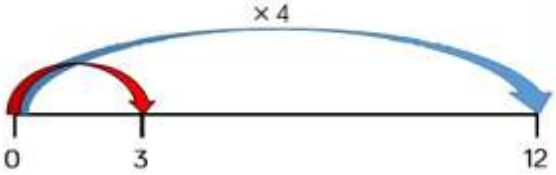
When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.
When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

Number Lines (blank)



A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

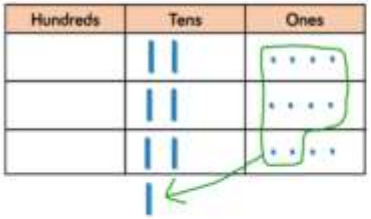
Benefits

Children can use blank number lines to represent scaling as multiplication or division.

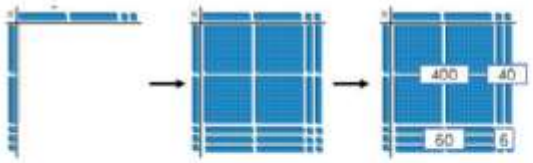
Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.

Base 10/Dienes (multiplication)



$$\begin{array}{r} 24 \\ \times \quad 3 \\ \hline 72 \\ \hline 1 \end{array}$$



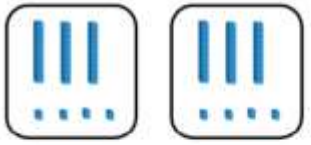
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

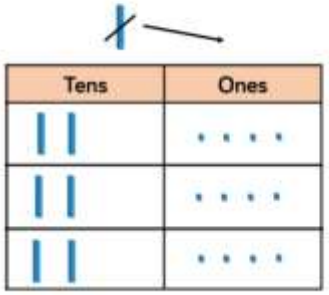
As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.

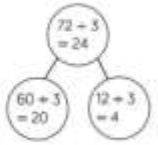
Base 10/Dienes (division)



$68 \div 2 = 34$



$72 \div 3 = 24$



Benefits

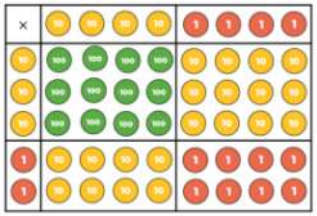
Using Base 10 or Dienes is an effective way to support children's understanding of division.

When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

Place Value Counters (multiplication)

$$\begin{array}{r} 34 \\ \times\ 5 \\ \hline 170 \\ \hline 1\ 2 \end{array}$$



$$\begin{array}{r} 44 \\ \times\ 32 \\ \hline 8 \\ 80 \\ 120 \\ +\ 1200 \\ \hline 1408 \end{array}$$

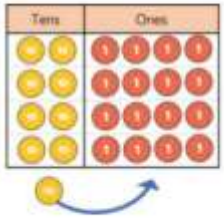
Benefits

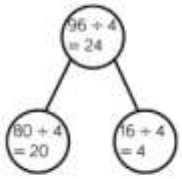
Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

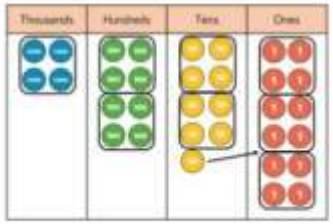
As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.

Place Value Counters (division)







$$\begin{array}{r} 1223 \\ 4\overline{)\,4892} \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of division.

When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.

TIMES TABLES

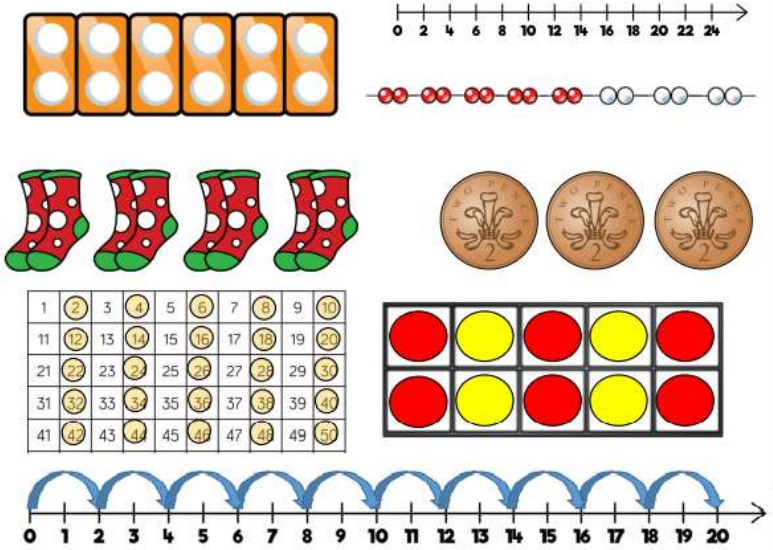
Skill	Year	Representations and models	
Recall and use multiplication and division facts for the 2-times table	2	Bar model Number shapes Counters Money	Ten frames Bead strings Number lines Everyday objects
Recall and use multiplication and division facts for the 5-times table	2	Bar model Number shapes Counters Money	Ten frames Bead strings Number lines Everyday objects
Recall and use multiplication and division facts for the 10-times table	2	Hundred square Number shapes Counters Money	Ten frames Bead strings Number lines Base 10

Skill	Year	Representations and models	
Recall and use multiplication and division facts for the 3-times table	3	Hundred square Number shapes Counters	Bead strings Number lines Everyday objects
Recall and use multiplication and division facts for the 4-times table	3	Hundred square Number shapes Counters	Bead strings Number lines Everyday objects
Recall and use multiplication and division facts for the 8-times table	3	Hundred square Number shapes	Bead strings Number tracks Everyday objects
Recall and use multiplication and division facts for the 6-times table	4	Hundred square Number shapes	Bead strings Number tracks Everyday objects

Skill	Year	Representations and models	
Recall and use multiplication and division facts for the 7-times table	4	Hundred square Number shapes	Bead strings Number lines
Recall and use multiplication and division facts for the 9-times table	4	Hundred square Number shapes	Bead strings Number lines
Recall and use multiplication and division facts for the 11-times table	4	Hundred square Base 10	Place value counters Number lines
Recall and use multiplication and division facts for the 12-times table	4	Hundred square Base 10	Place value counters Number lines

Skill: 2 times table

Year: 2



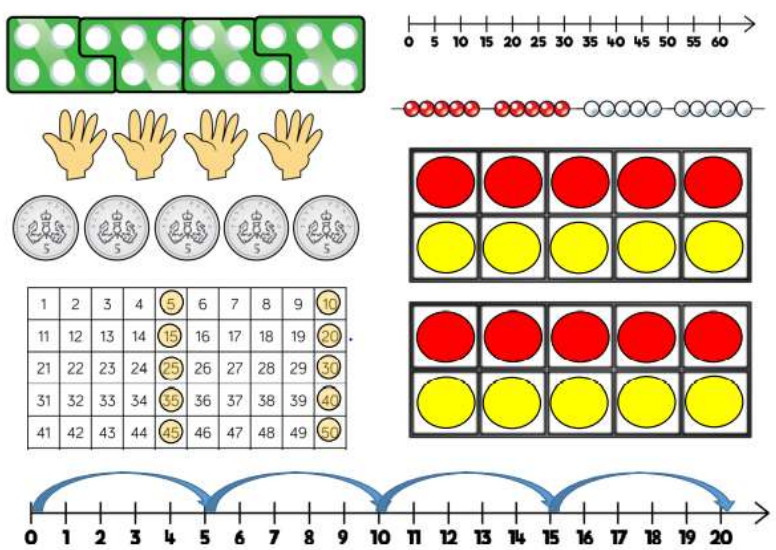
Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the two times table, using concrete manipulatives to support. Notice how all the numbers are even and there is a pattern in the ones.

Use different models to develop fluency.

Skill: 5 times table

Year: 2

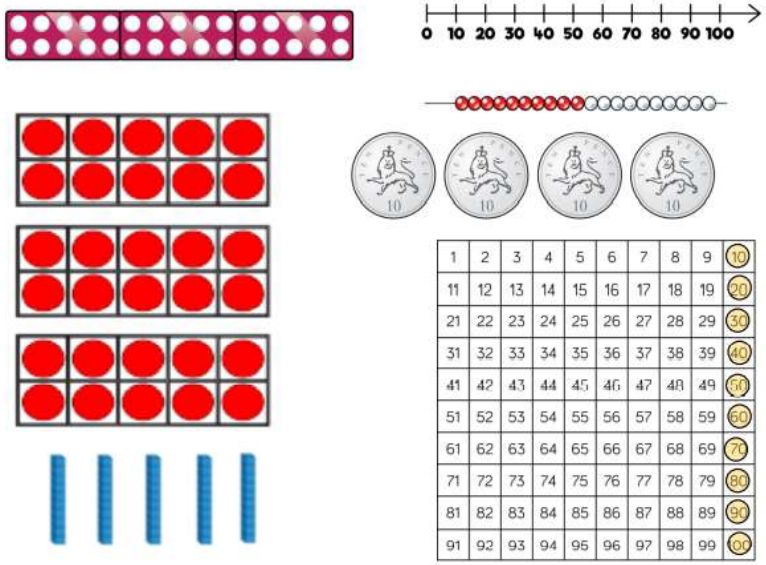


Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the five times table, using concrete manipulatives to support. Notice the pattern in the ones as well as highlighting the odd, even, odd, even pattern.

Skill: 10 times table

Year: 2

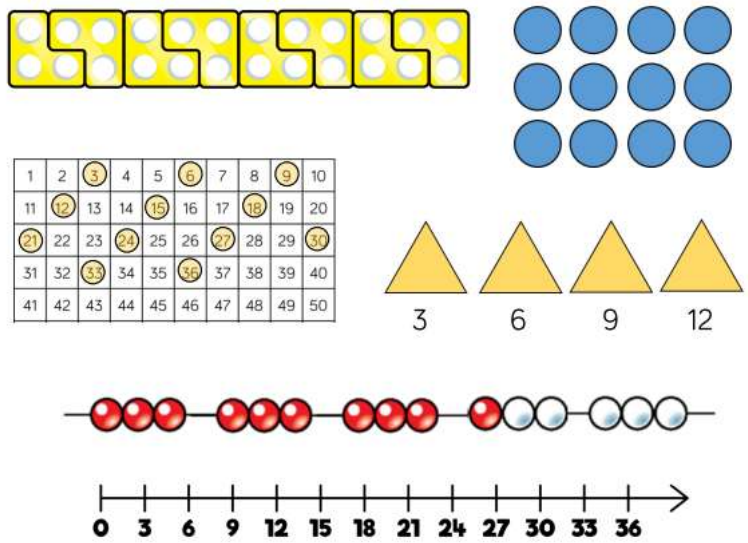


Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the ten times table, using concrete manipulatives to support. Notice the pattern in the digits- the ones are always 0, and the tens increase by 1 ten each time.

Skill: 3 times table

Year: 3

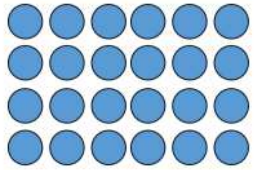


Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the three times table, using concrete manipulatives to support. Notice the odd, even, odd, even pattern using number shapes to support. Highlight the pattern in the ones using a hundred square.

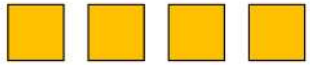
Skill: 4 times table

Year: 3

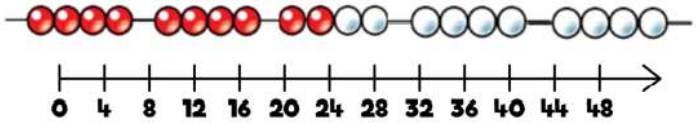


1	2	3	(4)	5	6	7	(8)	9	10
11	(12)	13	14	15	(16)	17	18	19	(20)
21	22	23	(24)	25	26	27	(28)	29	30
31	(32)	33	34	35	(36)	37	38	39	(40)
41	42	43	(44)	45	46	47	(48)	49	50

4	8	12	16	20
24	28	32	36	40
44	48	52	56	60



4 8 12 16



0 4 8 12 16 20 24 28 32 36 40 44 48

Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the four times table, using manipulatives to support. Make links to the 2 times table, seeing how each multiple is double the twos. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

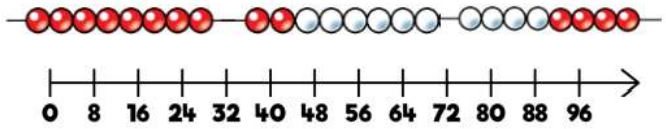
Skill: 8 times table

Year: 3

8 16 24 32

8	16	24	32	40
48	56	64	72	80

1	2	3	4	5	6	7	(8)	9	10
11	12	13	14	15	(16)	17	18	19	20
21	22	23	(24)	25	26	27	28	29	30
31	(32)	33	34	35	36	37	38	39	(40)
41	42	43	44	45	46	47	(48)	49	50
51	52	53	54	55	(56)	57	58	59	60
61	62	63	(64)	65	66	67	68	69	70
71	(72)	73	74	75	76	77	78	79	(80)
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

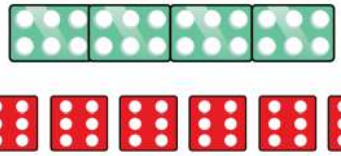


0 8 16 24 32 40 48 56 64 72 80 88 96

Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the eight times table, using manipulatives to support. Make links to the 4 times table, seeing how each multiple is double the fours. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

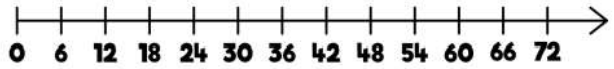
Skill: 6 times table

Year: 4



1	2	3	4	5	(6)	7	8	9	10
11	(12)	13	14	15	16	17	(18)	19	20
21	22	23	(24)	25	26	27	28	29	(30)
31	32	33	34	35	(36)	37	38	39	40
41	(42)	43	44	45	46	47	(48)	49	50
51	52	53	(54)	55	56	57	58	59	(60)
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

6	12	18	24	30
36	42	48	54	60
66	72	78	84	90



0 6 12 18 24 30 36 42 48 54 60 66 72

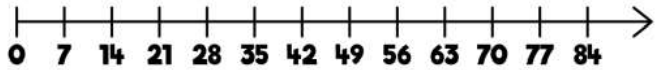
Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the six times table, using manipulatives to support. Make links to the 3 times table, seeing how each multiple is double the threes. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

Skill: 7 times table

Year: 4

7	14	21	28	35
42	49	56	63	70

1	2	3	4	5	6	(7)	8	9	10
11	12	13	(14)	15	16	17	18	19	20
(21)	22	23	24	25	26	27	(28)	29	30
31	32	33	34	(35)	36	37	38	39	40
41	(42)	43	44	45	46	47	48	(49)	50
51	52	53	54	55	(56)	57	58	59	60
61	62	(63)	64	65	66	67	68	69	(70)
71	72	73	74	75	76	(77)	78	79	80
81	82	83	(84)	85	86	87	88	89	90
(91)	92	93	94	95	96	97	(98)	99	100



0 7 14 21 28 35 42 49 56 63 70 77 84

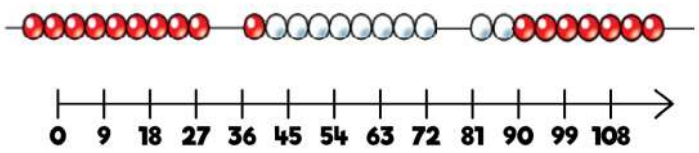
Encourage daily counting in multiples both forwards and backwards, supported by a number line or a hundred square. The seven times table can be trickier to learn due to the lack of obvious pattern in the numbers, however they already know several facts due to commutativity. Children can still see the odd, even pattern in the multiples using number shapes to support.

Skill: 9 times table

Year: 4

9	18	27	36	45
54	63	72	81	90

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



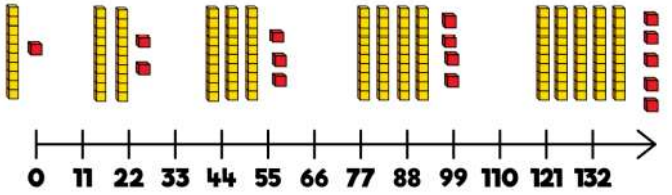
Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the nine times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support as well as noting the odd, even pattern within the multiples.

Skill: 11 times table

Year: 4

11	22	33	44	55	66
77	88	99	110	121	132

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



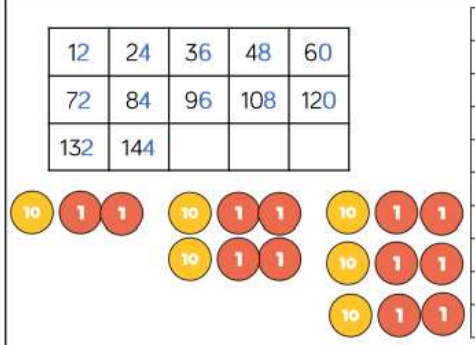
Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the eleven times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support. Also consider the pattern after crossing 100

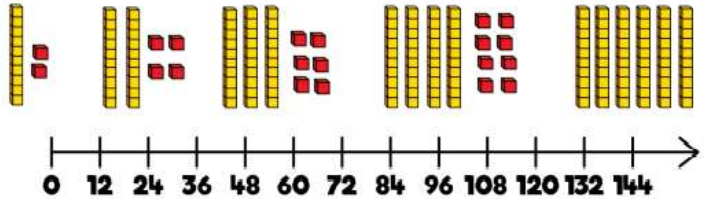
Skill: 12 times table

Year: 4

12	24	36	48	60
72	84	96	108	120
132	144			



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



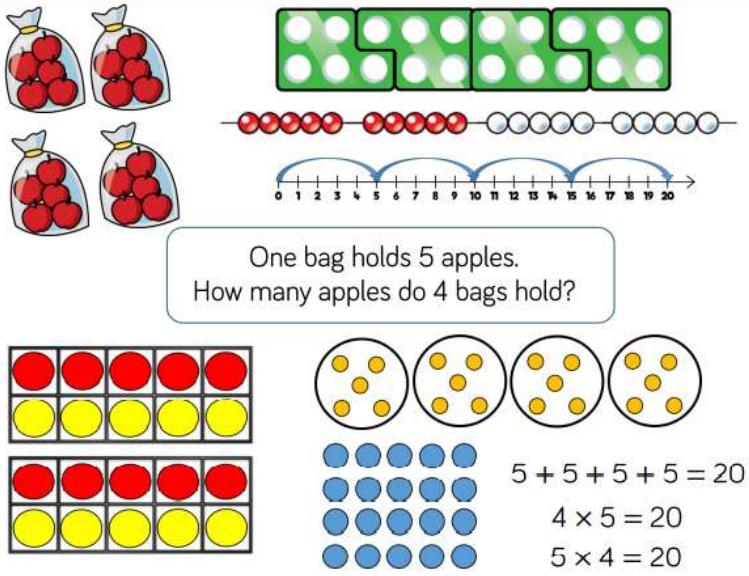
Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the 12 times table, using manipulatives to support. Make links to the 6 times table, seeing how each multiple is double the sixes. Notice the pattern in the ones within each group of five multiples. The hundred square can support in highlighting this pattern.

MULTIPLICATION 

Skill	Year	Representations and models	
Solve one-step problems with multiplication	1/2	Bar model Number shapes Counters	Ten frames Bead strings Number lines
Multiply 2-digit by 1-digit numbers	3/4	Place value counters Base 10	Short written method Expanded written method
Multiply 3-digit by 1-digit numbers	4	Place value counters Base 10	Short written method

Skill: Solve 1-step problems using multiplication

Year: 1/2



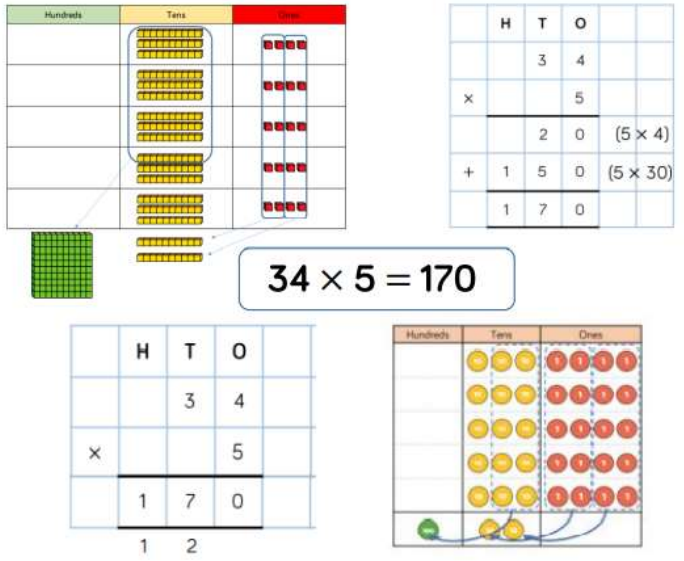
Children represent multiplication as repeated addition in many different ways.

In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally.

In Year 2, children are introduced to the multiplication symbol.

Skill: Multiply 2-digit numbers by 1-digit numbers

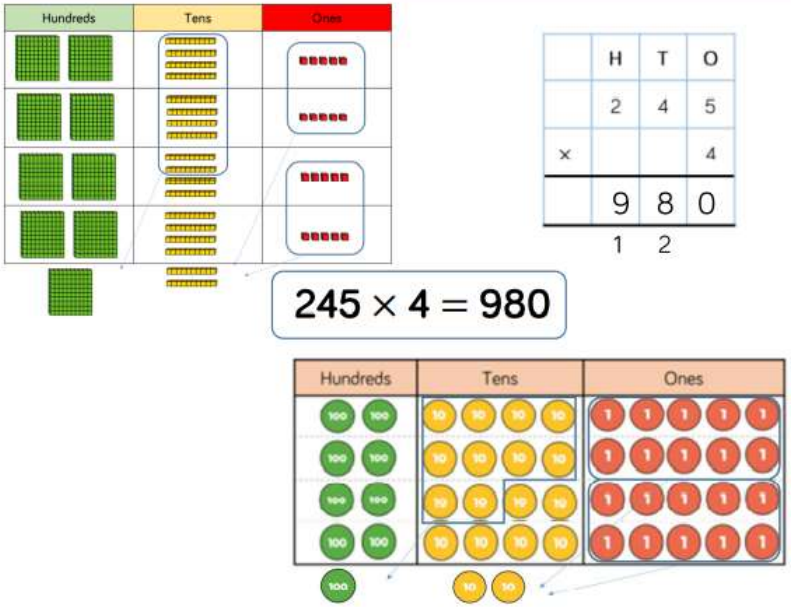
Year: 3/4



Teachers may decide to first look at the expanded column method before moving on to the short multiplication method.
The place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.

Skill: Multiply 3-digit numbers by 1-digit numbers

Year: 3/4



When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short, formal written method.
Base 10 and place value counters continue to support the understanding of the written method.
Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.

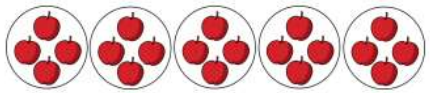
DIVISION 

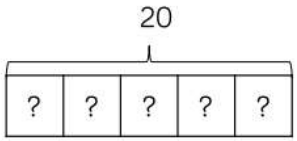
Skill	Year	Representations and models	
Solve one-step problems with division (sharing)	1/2	Bar model Real life objects	Arrays Counters
Solve one-step problems with division (grouping)	1/2	Real life objects Number shapes Bead strings Ten frames	Number lines Arrays Counters
Divide 2-digits by 1-digit (no exchange sharing)	3	Straws Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (sharing with exchange)	3	Straws Base 10 Bar model	Place value counters Part-whole model

Skill	Year	Representations and models	
Divide 2-digits by 1-digit (sharing with remainders)	3/4	Straws Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division
Divide 3-digits by 1-digit (sharing with exchange)	4	Base 10 Bar model	Place value counters Part-whole model
Divide 3-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division

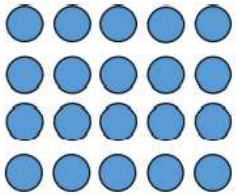
Skill: Solve 1-step problems using multiplication (sharing)

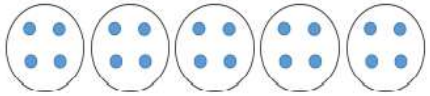
Year: 1/2





There are 20 apples altogether.
They are shared equally between 5 bags.
How many apples are in each bag?





$20 \div 5 = 4$

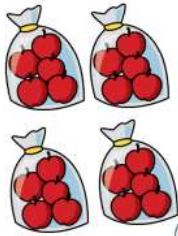
Children solve problems by sharing amounts into equal groups.

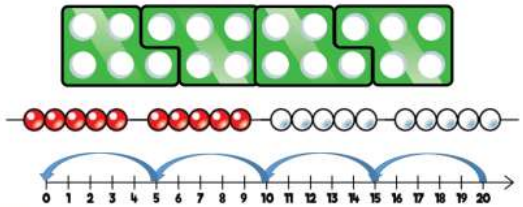
In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally.

In Year 2, children are introduced to the division symbol.

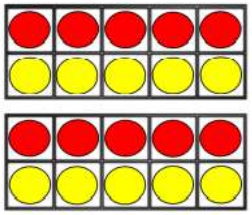
Skill: Solve 1-step problems using division (grouping)

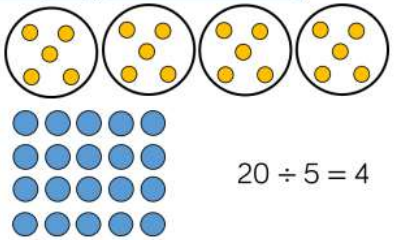
Year: 1/2





There are 20 apples altogether.
They are put in bags of 5.
How many bags are there?



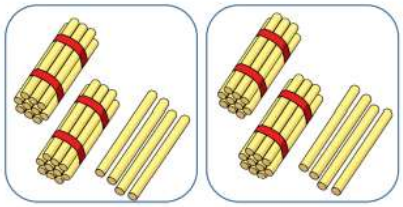


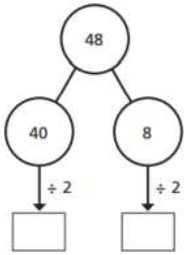
$20 \div 5 = 4$

Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division.

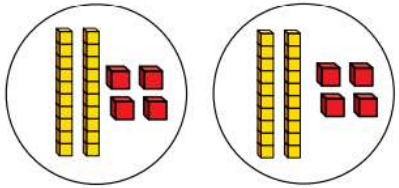
Skill: Divide 2-digits by 1-digit (sharing with no exchange)

Year: 1/2





$48 \div 2 = 24$



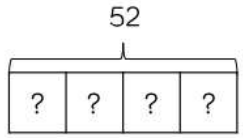
When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones.

Straws, Base 10 and place value counters can all be used to share numbers into equal groups.

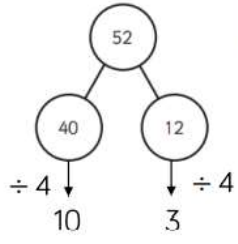
Part-whole models can provide children with a clear written method that matches the concrete representation.

Skill: Divide 2-digits by 1-digit (sharing with exchange)

Year: 3/4



$52 \div 4 = 13$



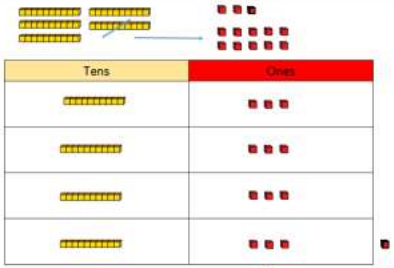
$10 + 3 = 13$

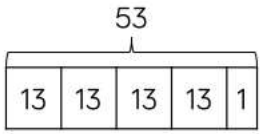
When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones. Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

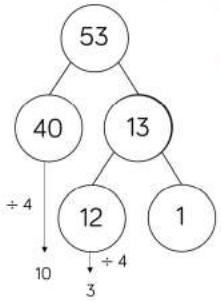
Flexible partitioning in a part-whole model supports this method.

Skill: Divide 2-digits by 1-digit (sharing with remainders)

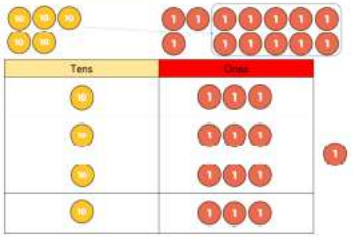
Year: 3/4







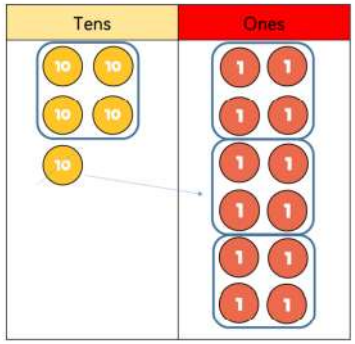
53 ÷ 4 = 13 r1

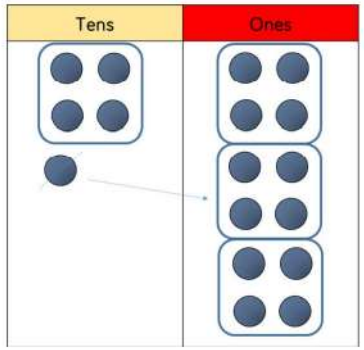


When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones.
Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made.
Flexible partitioning in a part-whole model supports this method.

Skill: Divide 2-digits by 1-digit (grouping)

Year: 4/5





52 ÷ 4 = 13

When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor.

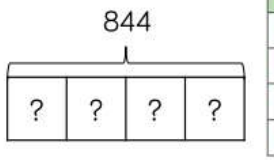
Language is important here. Children should consider 'How many groups of 4 tens can we make?' and 'How many groups of 4 ones can we make?'

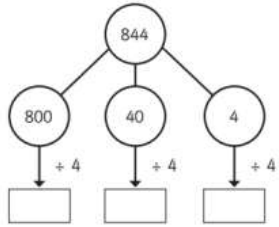
Remainders can also be seen as they are left ungrouped.

Skill: Divide 3-digits by 1-digit (sharing)

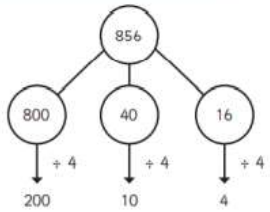
Year: 4

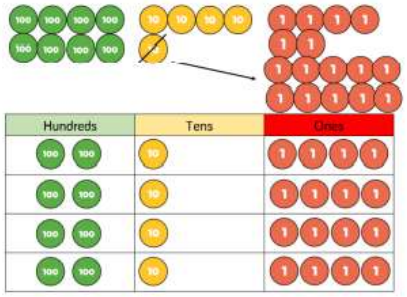
844 ÷ 4 = 211





844 ÷ 4 = 211





Children can continue to use place value counters to share 3-digit numbers into equal groups.
Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows.
This method can also help to highlight remainders.
Flexible partitioning in a part-whole model supports this method.

Glossary

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Numbers can be multiplied in any order.

Dividend – In division, the number that is divided.

Divisor – In division, the number by which another is divided.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make a product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Quotient – The result of a division

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor